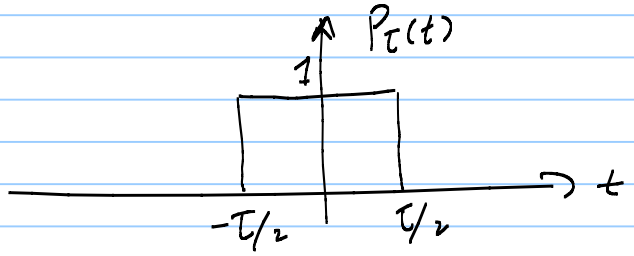
Fourier Transform

Note Title 2/28/2019

$$X(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$



$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$X(w) = \int_{-\tau/2}^{\tau/2} 1 \cdot e^{-jwt}\, dt = \frac{1}{-jw} e^{-jwt} \Big|_{-\tau/2}^{\tau/2}$$

$$= \frac{1}{-jw}\left[e^{-j\frac{w\tau}{2}} - e^{j\frac{w\tau}{2}} \right]$$

$$= \frac{2}{w}\left[\frac{e^{j\frac{w\tau}{2}} - e^{-j\frac{w\tau}{2}}}{2j} \right] = \frac{2}{w}\sin\left(\frac{w\tau}{2}\right)$$

$$P_\tau(t) \longleftrightarrow \frac{2}{w}\sin\left(\frac{w\tau}{2}\right)$$

$\text{sinc}(x) = ?$

$$\text{sinc}(aw) = \frac{1}{a\pi w}\sin(a\pi w)$$

$$a = \frac{\tau}{2\pi}$$

$$\text{sinc}\left(\frac{w\tau}{2\pi}\right) = \frac{2}{w\tau}\sin\left(\frac{w\tau}{2}\right)$$

OR $\quad \tau\,\text{sinc}\left(\frac{w\tau}{2\pi}\right) = \frac{2}{w}\sin\left(\frac{w\tau}{2}\right)$

$$P_\tau(t) \longleftrightarrow \tau\,\text{sinc}\left(\frac{w\tau}{2\pi}\right)$$